GEOMETRY, MATHEMATICS IN SECONDARY SCHOOLS.[1]

THE study of algebra is interesting, and the results are very gratifying to the teacher, but the most interesting work in secondary mathematics is the work in geometry.

Here we can actually see the development of the class if it has received proper treatment, but with improper care and formal recitation, the results may be very different. Instead of wide-awake faces always on the *qui vive* and constantly on the lookout for new truth, you may find a listless, monotonous conduct that will dampen the ardor of a most enthusiastic teacher. But pupils generally take on some of the qualities and characteristics of their teachers. Fortunate the class which has an instructor with enough life to sometimes get beyond the time-beaten paths of the formal recitation, and who is ready to ask more of the pupil than a reproduction of the demonstrations that have been handed down to us through twenty-five centuries.

Present indications point to a radical departure from the old plan of having the pupil "ride his pony" through the demonstrations of our present text-books.

The demand is for a book that will require the pupil to do some thinking on his own account. A book that will be to the pupil a guide in the more difficult work, suggesting the steps in the proof by questions rather than giving the entire demonstrations as nearly all of our books now do.

At least one book has been arranged on this plan, but while it is a great step in the right direction it is very liberal with its helps.

Under such a method the study of geometry, with a consecrated teacher, would give to the pupil much of that discipline which is usually attributed to it, but which is too often lost in the formal memory work of the old plan.

[1] Portion of a paper on "Mathematics in Secondary Schools."

Yet with the books we have, much can be done by the teacher toward correcting or rather avoiding the faults and difficulties arising from their use. Greatest care should be taken in the study of the definitions, as they are usually collected on the first few pages of the text. While the language given is better than any the pupil might use, and they cover the case better than the pupil would do it even after he understands the meaning of the definition given, it is well for him to give in his own language what seems to him to cover the case, and let the teacher point out the faults if there be any, thus teaching the value of words used and the great care necessary in giving expression to mathematical ideas. Time thus used is never wasted. We can well afford to spend plenty of time talking with our classes about these same definitions, explaining away the difficulties before we require our pupils to memorize them.

One glaring instance in my own experience always comes before me when I am working on the geometric definitions.

The definition for a "construction" as I was required to learn it, was "a graphical representation of a geometric conception." It meant nothing to me. I well remember how I was always afraid of that word "construction," and some others that were almost as bad, because we had no explanation. A lack of understanding here makes it hard sledding throughout the entire year.

The definitions, axioms, theorems, and corollaries must be learned and learned well, but the pupil should never be required to memorize them until he understands the meaning so well that they present to his mind a mental picture with all of the details, and it is the teacher's business to see that he has such understanding.

We should not be afraid of explanation, especially at the very beginning of the work. Take the concrete case and work to the general.

We must not pay too much attention to some of our teachers of pedagogy who say that it is a poor teacher and a poor class that has to resort to the concrete case. The general will follow the particular, and the pupil will learn to generalize

only after he understands the particular cases that go to make up a part of the general case.

The first ten weeks necessarily means most careful work on the part of the teacher; watch the pupil closely and do not think that because he does not ask questions he understands. We must teach him to ask questions. Get it out of his head that he is to impress you with his knowledge of the matter at hand; rather teach him to impress you with a frank, investigating nature. Make him understand that it is not necessary to work for marks, as the marks are capable of looking out for themselves if he has the proper spirit. Complaint is continually made that pupils insist on memorizing the demonstrations given in the book. May not the teacher sometimes be at fault here? Does the pupil know how to study? Have we been careful enough in showing him the proper plan of attack? Early in the year do we not sometimes insist that the pupil shall learn the proofs well in order that he may be able to apply the principles to later work? Have we studied the first few theorems with the pupils or have we left them to struggle for themselves?

Some of the pupils will not know *how* or *what* to do unless they have the help of the teacher. Some of them will invariably commit to memory the work done in the book. We should not criticise them too harshly, because that is probably the only kind of studying they have been taught to do. It is better for the class to study the first few theorems and demonstrations in the recitation period, under the direction of the teacher, and the teacher should take some time each day for sight work on the advance lesson.

A few well chosen questions will sometimes do the pupil more real good than an hour of hard study.

The first thing to be done is to get a perfectly clear understanding of the theorem, analyze it, and know just what is required, and how much may be taken as the basis for work. If the pupil then fails to see the method of proof, let him assume the truth of the conclusion, and examine the relations back to the hypothesis. The class will soon get the idea of what the work in geometry means, and some of the better minds will be

attempting the solution of theorems as exercises, and will show decidedly more interest in a page of exercises than in the regular propositions.

Another plan that will add strength and power of imagination, is the demonstration of theorems without a figure to look at. Besides giving them power to picture conditions in the mind, it teaches them to concentrate their thought, and they will be able to talk intelligently on an exercise or problem without the aid of pencil and paper.

When a boy or girl fifteen years of age sits down, and referring only to a mental picture, describes the process of determining the position of a point, or of finding the locus of a point under given conditions, and is able to see the exception, if there be one, that boy or girl will be no weakling in mathematics. The tendency to memorize will certainly give way to such training.

The careless manner in which some teachers pass over the subject of *loci* is to be lamented and condemned. It seems sometimes as if it were a case of "blind leading the blind." No one principle in geometry is more interesting or more valuable, and nothing does more toward eliminating the question of chance in our work in originals. Some teachers do not touch it at all. Some merely draw a figure and tell the class that it is true, for anybody with eyes can see it.

Again some pupils get the idea that the locus of a point is entirely independent of the conditions in the case, and whenever the term locus is met with *it is a perpendicular to a line at its middle point.* How much of mathematical value in such teaching? It takes time and numerous exercises showing the almost unlimited number of cases arising, each under its own conditions, to impress the pupil with the true meaning of the *locus of a point.* But it is worth all the time it takes, for when the pupil begins his exercises in construction he has the one instrument that insures his success.

And what shall we do with the originals, as they are called? Work them; they are the flesh and blood that give beauty and life to the framework of geometry. Work all that time will

possibly permit. The class that is well grounded in the theorems and demonstrations, in axioms and definitions, that has had some sight work and work from mental pictures, will handle originals in such a way as to bring joy to the heart of the teacher.

How shall the pupil approach the original exercise? By the natural method, Plato's method, the method that has been used for more than two thousand years. Approach it as the scientist approaches his problem, assume the conditions as true, and go back over the ground and see if you can find a chain of relations that will lead to the proof of the assumption. Give a bright boy the task of finding the locus of a point under given conditions. He will construct one, two, or three points; he sees, perhaps, that it is not a straight line, he assumes that it is a curve, and as he knows only one curve, he will try to prove his assumption, that the locus is the circumference of a given circle. It is the natural method.

The teacher must always be critical if he wishes to find the same qualities in his class. He must insist on the use of correct mathematical expressions till they become a part of the pupil. To obtain the best results, the recitation should be an informal discussion, and that is possible only when the teacher gets so close to the pupil that perfect frankness exists between them, and each enjoys the confidence and respect of the other. Then the teacher's presence will stimulate free discussion rather than reserve, and the pupil will be ready to uphold his opinions against those of the teacher in such a way as to carry conviction with his argument. Both teacher and pupil must feel that they have no right to make a statement to the class that they cannot prove. The "omnipresent why" should never be allowed to escape the pupil's attention.

What shall we do to keep our pupils from *wandering* in their demonstrations? Insist on their knowing their destination and their route. Most wandering arises from a lack of care in studying or analyzing the theorem. They do not know the hypothesis and conclusion sufficiently well to trace a line of thought from one to the other, even though they may see certain relations.

The teacher should not forget to teach the history of mathematics along with the theory. The interest of the class may be greatly increased if he will tell them some historical fact or anecdote along with the solution of the problem. Call the attention of the class to the beauty of the theorems that form the groundwork of geometry, especially to such as "The sum of the three angles of a triangle is equal to two right angles," and "The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides."

Tell the pupils of the discovery of such principles; show them how the Egyptians 3400 years ago used three pegs and a rope to show the relation of the sides and hypotenuse, and thus constructed the right angle; show them the mistake of these same Egyptians in computing the areas of parallelograms and and trapezoids; tell them that Thales, five centuries B. C., discovered that every angle inscribed in a semicircle is a right angle, and that the sum of the three angles of a triangle is equal to two right angles.

It will interest them to know that Plato went to Egypt and studied mathematics and was so deeply interested that he had placed over his door the following inscription: "Let no one who is unacquainted with geometry enter here." Tell them Euclid's answer to the first Ptolemy, 300 B. C., when Ptolemy asked if there was no easier way to learn geometry than by studying the elements as prepared by Euclid; Euclid answered: "There is no royal road to geometry." Tell them that Euclid's elements with some additions has been used as a text-book for more than two thousand years.

They will listen closely to the story of the life and death of Archimedes at Syracuse. Tell them how nearly he approximated the value of π, and how since his time "circle squarers" by the score have wasted their energies. These incidents and others of the same sort, told when the class is studying some of the related theorems will add much to their interest in the work.

I once read a list of answers to questions relative to reforms in teaching mathematics. One said: "Get teachers who know more." We would better add to this, *and know better how to*

teach. The work in our secondary schools must not only give the pupils preparation for college, but must give to that large class who go no farther, the best possible preparation for life's duties; but to me, the mathematics for these two classes in secondary schools does not need to be differentiated. What is best in preparation for college mathematics is the best training the mind can get for an active business career. We may not need *limits* and *loci* in business, but the mind that has been concentrated on these principles in school is better fitted for concentration on public works and corporation law.

The twofold purpose of the school should be kept constantly in mind, and whatever work is done, should be done with such a degree of accuracy in mathematical thought and expression that the training the pupil gets will noticeably affect his work in other lines.

While accuracy of expression and careful reading are necessary in algebraic solutions and formulæ, geometry has the greater effect in ordering the mind of the pupil. No other secondary study pursued for one year will give such correct habits of thought and such careful use of words. This same accuracy of expression may cling to the individual throughout his whole life. The pupils study should give him both a proper basis on which to build his future work in mathematics, and the power of ready help in the everyday questions of life.

It should also give him a taste for proper argument, founded on fact and substantiated at every turn by unassailable proofs. Yet to my mind the greatest good to be derived from the study is the "ethical good;" it should teach him the law of order; it should give him integrity of purpose and the power of concentration; it should give him careful, yes, honest expression for all his ideas; it should give him the ability to analyze the questions of life, and separate the good from the bad, the vital from the accompaniments, and to recognize and use the best means at hand for his own good, and happiness.

All these things should be kept constantly in mind by the teacher, and on every possible occasion he should impress them upon his pupils. In other words, humanize the science of mathematics.

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